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AUTHOR Gordon, Myles
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ABSTRACT

This paper examines a range of studies done on the "Geometric Supposer" software and its use since the mid-1980s. It describes how the "Geometric Supposer" is a response to the call for reform by the mathematics community to create classroom environments that allow students to construct their own knowledge. This approach to learning mathematics is called "guided inquiry" and is described and contrasted with current practice. Research findings about students and teachers from studies using the "Geometric Supposer" are presented. Findings are reported about mathematical behaviors, attitudes, geometry content, learning behaviors, and teaching behaviors. Key issues and dilemmas that arise in the course of using the software with a pedagogy of guided inquiry are also identified. Since the "Supposer" exemplifies mathematics education as called for in the prescriptions for change by the National Council of Teachers of Mathematics and the Mathematical Sciences Education Board, the paper argues that what has been learned from six years of "Supposer" experience can serve as a guide for thinking about mathematics education reform beyond geometry. (MDH)

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What Is the *Geometric Supposer* a Case Of?

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This paper examines a range of studies done on the *Geometric Supposer* software and its use since the mid-1980s. It summarizes the findings of these studies of student learning, teacher attitudes, school contexts, and implementation. It identifies key issues and dilemmas that arise in the course of using the software with a pedagogy of guided inquiry. Since the *Supposer* exemplifies mathematics education as called for in the prescriptions for change by the National Council of Teachers of Mathematics and the Mathematical Sciences Education Board, the paper argues that what has been learned from six years of *Supposer* experience can serve as a guide for thinking about mathematics education reform beyond geometry.

First student: "In class, it's like 'This is how it is!'"

Second student: "In lab, it's like 'How is it?'"

"I see a lot of the way I learn in using the *Supposer*. . . . You learn from your observations. . . . I like learning from experience or from seeing what other people have done. That's what it is. You're doing the stuff yourself, so you're rediscovering stuff. It's really your own learning."

A Coherent Call for Change and a Model of an Answer

Amidst the clamor for education reform, the most coherent proposals have come from the mathematics education community.

The federal government, foundations, corporations, professional organizations of mathematicians and math educators, and universities — all agree that mathematics education as practiced in most U.S. schools is not meeting anyone's needs. They further agree that our inability to promote effective learning of mathematics for all students is a dangerous state of affairs. It limits the participation of a large portion of the population in the economic, political, and social mainstream of American life; it places the U.S. economy at a severe competitive disadvantage; and, some would say, it threatens the very security of the country.

What is the problem?

The National Council of Teachers of Mathematics (NCTM), in its recently released curriculum and evaluation standards document, puts the problem quite simply: "What a student learns depends to a great degree on *how* he or she has learned it" (1989, p. 5).

If what you reap depends on both *what* and *how* you sow, then we get every bit of mathematics learning that we deserve. Throughout their elementary and secondary schooling, we expect students to go through only the motions of mathematics, and perhaps more so than in other subjects, as John Goodlad (1984) found, what goes on in the mathematics classroom is teacher lecture and student recitation. It should come as no surprise then that what passes for learning in mathematics is the predigested mechanics of symbol manipulation. Students come away with a meager understanding of mathematical concepts and no first hand experience of the power and excitement of mathematics.

Is there an alternative?

Again, the NCTM (1989) provides a useful definition of mathematical literacy, embodied in five goals for students in mathematics education:

1. Learning to Value Mathematics
2. Becoming Confident in One's Own Ability

3. Becoming a Mathematical Problem Solver
4. Learning to Communicate Mathematically
5. Learning to Reason Mathematically

If this is "what" students should be learning, then where can we find the appropriate "how"?

Consensus is also emerging on that question, building on research findings in cognitive science on the nature of knowledge and knowledge making. These findings suggest what is often referred to as "a constructivist approach": "A close consideration of recent research on mathematical cognition suggests that in mathematics as in reading, successful learners understand the task to be one of *constructing meaning*, of doing interpretive work rather than routine manipulations" (Resnick, 1987, p. 12).

This approach is echoed in the blueprints for reform. The NCTM (1989) emphasizes that "'knowing' mathematics is 'doing' mathematics" (p. 7); the Mathematical Sciences Education Board (MSEB) of the National Research Council, in its report *Everybody Counts* (1989), points out that "students learn mathematics well only when they *construct* their own mathematical understanding" (p. 58).

If the verbs "lecture" and "recite" characterize most current mathematics instruction, the constructivist approach calls for a new set of verbs to characterize what ought to happen in the classroom. In the words of the National Research Council's MSEB (1989), if students are "to understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: 'examine,' 'represent,' 'transform,' 'solve,' 'apply,' 'prove,' 'communicate'" (pp. 58-59).

Moreover, this new approach calls for a shift in the perception that the learning of mathematics is an activity students ought to engage in only as isolated individuals. The National Research Council's MSEB (1989) calls for "genuine give-and-take in the mathematics classroom, both among students and between students and teachers. . . . This happens most readily when students work in groups, engage in discussion, make presentations, and in other ways take charge of their own learning" (p. 59). The NCTM (1989) argues that "Instruction should . . . include opportunities for appropriate project

work; group and individual assignments; discussion between teacher and students and among students . . ." (p. 10).

In many ways the experience of thousands of students and teachers over the past several years with the *Geometric Supposer* is consonant with the proposed new approach to mathematics education. In fact, I would like to argue that software like the *Supposer*, along with a rich collection of open-ended projects and problems, approached through a pedagogy of guided inquiry, is a model of how we can, as a community of educators, make real the vision that the NCTM and the MSEB have put before us.

The *Geometric Supposer* (Schwartz and Yerushalmy, 1984) is a series of software programs designed to introduce experimentation, inductive reasoning, and conjecturing into the learning of geometry in elementary and secondary schools.¹ The series consists of four programs. Three deal with a family of geometric shapes (*Triangles*, *Quadrilaterals*, *Circles*) and allow users to make on the computer screen any construction that can be carried out with a straightedge and compass on a random shape or a shape of their own making. The fourth, the *preSupposer*, is designed specifically to introduce younger students to geometric shapes, constructions, and definitions.

The software also includes supporting procedures with which users can make measurements on any element in the construction; can add, subtract, multiply, divide, or square those measurements; can rescale; and can return to previous shapes with the same or different constructions. Perhaps the most important feature of the software is that it allows users to repeat constructions as procedures on other shapes. With these features, the *Supposer* is a tool that enables students to explore the properties of shapes and geometric elements and to investigate whether the properties and consequences of a given construction on a given shape are dependent on some particular property of that shape, or whether the result can be generalized.

My argument about the relevance of the *Supposer* to mathematics education reform hinges on the findings of a yearlong study of *Supposer* use in three Boston-area schools that I conducted with Michal Yerushalmy and Daniel Chazan

(Yerushalmy, Chazan, & Gordon, 1987) and more than a dozen other research studies done on the *Supposer* over the past five years. These include studies of student learning, teacher attitudes and behaviors, school contexts, and implementation. I shall review some of these findings to support the contention that the *Supposer* experience might serve as a model for the national reform of mathematics education. Much of the work I will refer to is reported in a forthcoming book on the *Supposer* (Schwartz, Yerushalmy, & Wilson, in press) and elsewhere in the research literature.

Not all of the research yields answers. Sometimes it brings to light new questions or questions whose importance was not fully appreciated. I shall try to discuss some of these findings as well.

Finally, beyond the argument that the *Supposer* experience might serve as a model for national reform in mathematics education lies the question of education reform in general. I shall also consider whether the *Supposer* experience affords any insight into that broader question.

Some Remarks about Guided Inquiry

In the mid-1980s, those of us involved in the development of the *Supposers* and in the first research studies found ourselves learning mathematics in the domain of geometry in a new and exciting way. In reflecting on this experience we realized that a new style of learning calls for a new style of teaching. Working with the *Supposer*, we evolved a pedagogic strategy that we call "guided inquiry," which contrasts sharply with much of current practice in the mathematics classroom.

[In guided inquiry,] the content of the curriculum is the same as [that in] a standard geometry course . . . [However,] rather than focusing only on deductive reasoning and proof, the guided inquiry approach calls for students to integrate inductive reasoning and empirical work with conceptual work in solving problems and devising proofs. . . . While new material is often introduced by teacher lecture . . . , guided inquiry emphasizes laboratory work and class discussion in which students take a more active and responsible role in the learning process. (Yerushalmy et al., 1987/1990, p. 7)

In the computer laboratory, students investigate problems designed to introduce, reinforce, or bring to light ideas, axioms, or theorems from the curriculum. Working with the *Supposers*, students, usually in pairs, investigate the problem, making constructions and recording numerical and visual data. From these data, they are typically asked to generate conjectures and then to support their conjectures with arguments and proof. Returning to the classroom, discussion periods focus on the sharing of student data, conjectures, and supporting arguments, building collective understanding from individual investigations.

Clearly there is a high degree of overlap between the process of guided inquiry as we have described it and the vision of a new mathematics education put forward by the NCTM and the MSEB.

Although it is logically possible to separate the use of the *Supposer* from a pedagogy of guided inquiry, all of the studies I wish to call upon looked at guided inquiry classrooms. If, on the basis of these studies, one wishes to make an argument about a model for mathematics education reform, then the *Supposer* experience in conjunction with a pedagogy of guided inquiry must be taken as the kernel of the model.

The Good News from the Research

Findings about Students

The findings about student behavior with the *Supposers* can be sorted into four categories: mathematical behaviors, attitudes, geometry content, and learning behaviors. Before I summarize what we found in each category, let me point out that these behaviors were seen in a wide range of students, at all ability levels, and in a variety of school systems. This is not to say that all of the behaviors were seen all the time in all students. Nor is it to say that the behaviors were a direct consequence of using the *Supposer*. We can report only that we saw these behaviors occurring frequently, often in students who were not regarded by their school systems and their teachers as mathematically able.

Mathematical behaviors. We found that students were able to:

- Discern patterns in both numerical and visual data.

- Make conjectures.
- Make generalizations from numerical data, visual data, and formal statements.
- Construct definitions, axioms, and theorems.
- Construct arguments, both formal and informal, and distinguish between the two.
- Move from the intuitive to the formal and back.
- Use diagrams and understand which features of a diagram were specific to a particular diagram and which were not.
- Reorganize visual fields.
- Offer counterexamples.
- Build on existing knowledge.

Attitudes. We found that students displayed a curiosity, no doubt long dormant, and a willingness to explore. In sharp contrast to the normal course of events in mathematics classrooms, students seemed willing to take intellectual risks. They also seemed willing to make mistakes, a particularly uncommon occurrence when mathematics is thought of as a domain in which there are unique right answers. Above all, they seemed to value new ideas and to recognize that there would be occasions when they themselves would be the originators of such ideas.

Geometry content. Students in *Supposer* classes learned as much geometry as the students in traditional classes did, as indicated by their performance on department geometry examinations. In addition, however, they seemed to develop an interest in and an ability to explore geometric ideas not normally encountered in the standard curriculum. They were able to consider generalizations across polygons. They used constructions as an aid in proof. In short, they became geometers — nimble ones for the secondary school setting.

An example will illustrate students' ventures beyond the commonplace and their ability to generalize. A geometric construct encountered early in the study of triangles is the median, a line drawn from the vertex of a triangle to the midpoint of its opposite side (see figure 1, page 5). In most geometry texts, the standard defini-

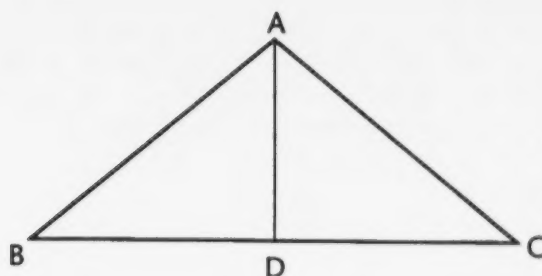


Figure 1

tion states that the median bisects the length of the opposite side. While exploring this construction with the *Geometric Supposer*, students discovered that the median also "bisects" the area of the triangle (that is, divides it into two triangles of equal area). The students tested this conjecture to see if it held for all triangles, saw that it did, and proceeded to investigate a proof as well. In some classes, students went on to consider whether this might be the basis of a definition of a median in a quadrilateral or even an n -gon, since there is no definitive "opposite side" in a quadrilateral.

Learning behaviors. Perhaps the most desirable outcome of the *Supposer* experience for many students was the growth of intellectual independence. They learned both to carry out independent investigations and to collaborate with one another when necessary. They learned to value their own ideas and those of others. This led to a growing ability to communicate about mathematics and to integrate individual and group learning. Above all, they assumed an increasing amount of responsibility for the making and transmitting of knowledge in the classroom.

Findings about Teachers

The research findings on teachers can be sorted in much the same way as the student findings into four categories: mathematical behaviors, attitudes, geometry content, and teaching behaviors. As in the case of the students, not all the findings apply to all of the teachers all of the time. Nonetheless, we have made the observations reported here across a wide range of teachers in a variety of schools, working with students of widely ranging abilities.

Mathematical behaviors. The most extraordinary observation we made about teachers' mathematical behavior in *Supposer* classrooms was the degree to which teachers started to "do" mathematics instead of simply "teaching" the subject. We believe that this modeling of desired behavior will surely influence in a strongly positive way the students in these teachers' classes (In fact, we found it to be one of the most important and powerful teaching strategies).

This "doing" of mathematics took several forms. In addition to working collaboratively with students on still unsolved problems, many of the *Supposer* teachers began to collaborate with fellow teachers in thinking about and making mathematics. To the extent that such collaboration occurs, it reflects two promising directions for professional development: (1) teachers seeing themselves as active members of the mathematics community, and (2) ongoing collegial exchange as an alternative to the injection of outside expertise as the key source for continuing professional growth.

Finally, many teachers reported excitement, liberation, and invigoration with the use of open-ended problems and with the utterance — with dignity and pride — of the words, "I don't know."

Attitudes. Among the attitude changes we observed in teachers was a growing willingness to explore new technologies and their potential for enhancing the teaching and learning of mathematics. Often we found teachers eager to enliven the teaching of the subject. Implicit in this attitude was a belief that lack of positive student response to the subject might be due, at least in part, to dull material presented in dull ways, and in ways that are ineffective for some students.

In addition, teachers working with the *Supposer* and guided inquiry displayed an uncommon eagerness to discern the varying learning styles of their students and to find ways to modify their teaching to address these varying styles.

Geometry content. The most remarkable finding in this category is that many of the *Supposer* teachers realized that the scope and sequence of the traditional geometry course was not sacred. They found it possible to think

about a wide variety of routes through even quite traditional material.

Further, many of the *Supposer* teachers came to a new understanding of the intertwined roles of theorem formulating and theorem proving. In traditional geometry classrooms, theorems originating in antiquity arrive via the text, and students set out to prove these theorems which they assume to be true. The emphasis in such classrooms is clearly on proof, to the neglect of the equally important mathematical act of formulating the theorem to be proven. In *Supposer* classrooms this imbalance is redressed in some measure. Over time many *Supposer* teachers learned how to perform an intricate and difficult intellectual minuet, moving with grace from inductive to deductive thinking and back.

Teaching behaviors. Clearly, *Supposer* teachers struggled to integrate technology into the curriculum, and to some extent they succeeded. But there were more important changes in the behaviors of these teachers. Many of them learned to "let go," to give up their role as the only authority in the classroom. They learned to acknowledge the value of students' ideas and to respond to them. In classrooms where little inquiry typically occurred, where collaboration might have been viewed as cheating, and where discussion was a rarity, *Supposer* teachers learned to support student inquiry, to foster collaboration among students, and to moderate the vigorous discussions that inevitably followed sessions in the computer lab.

In sum, teachers were able to go well beyond teacher talk and to support students well beyond recitation and mechanics. They not only learned how to support student inquiry, but, in many cases, they became inquirers themselves. As such, they became both members of and managers of a learning community.

I recognize that this has been an unrelievedly positive report of both student and teacher findings. Needless to say, the news is not all good. There were, and are, many problems not yet overcome. Indeed, there are many whose dimensions we are only now coming to appreciate. I shall now turn to that part of the *Geometric Supposer* story.

The Other News from the Research

The first and by far the most important of the other findings is that even with extensive, varied, and often individualized support, implementing the *Supposer* and guided inquiry is not easy. In one implementation project, roving advisors visited schools and worked with teachers as they tried to incorporate the *Supposer* into their curriculum and to modify their teaching styles in the direction of guided inquiry. In addition, more experienced teachers and members of the research team met with teachers in monthly seminars and provided a great deal of informal coaching. Computers were made available to teachers so that they could work on problems by themselves and plan lessons away from the pressures of school. Extensive written materials in the form of projects and problems were made available for teachers to use. And still it wasn't easy!

The difficulties derive from the changed and expanded demands on teachers, the dilemmas that teachers are confronted with, and the deep shifts in thinking about themselves and their subject that face teachers who attempt to implement this new approach to mathematics education.

Demands

Working with the *Supposer* in a guided inquiry mode takes more time and effort than traditional teaching, and a different kind of effort as well. In part, the greater time demands come from the need to deal with technology in the classroom. As computers come to permeate every facet of our society, this problem will likely diminish in importance, but for the moment it remains a difficulty.

A second kind of increased demand comes from the heavy emphasis on the deductive and formal aspects of the subject in the way most teachers have learned and taught. We have been asking them to shift their perspective on the subject and to begin to value and incorporate into their teaching inductive modes of thinking. Moreover, we have been asking them to consider seriously the idea that the sequence of topics that is covered in the traditional geometry course, either the one they previously taught or the one they were taught, is far more

arbitrary than they thought. In fact the sequence is set, in very large measure, by the text in use. If the pedagogic style becomes one of guided inquiry, then at any given time quite different topics are likely to come up for exploration and discussion.

I believe that the heart of the complaint about greater demands lies in the uncertainty teachers experience when they are not in a position to know precisely what issues, constructions, theorems, or even topics their students will raise. Teachers feel that if they are going to be responsive in this sort of open-ended inquiry environment, then they must be ready to respond to a range of potential questions that is far less predictable than would be the case in a traditional classroom. Under these circumstances they need more time to prepare and review for class.

Open-ended problems need to be explored, conjectures considered, and possible proofs identified before class. Grading homeworks and quizzes based on open-ended problems is the mathematical equivalent of dealing with a stack of essays: all are prompted by the same question, but each one requires careful consideration and a unique response. With time, no doubt, teachers will build up a repertoire of problems and strategies, and this effort will diminish, but it will never be as mechanical as correcting two-column proofs, or as intellectually empty as checking student responses, year after year, against the answer key in the teacher's edition.

A very thoughtful teacher in one of our studies had a successful year with the *Supposer* but then became disappointed with her students' performance and level of understanding the following year. Finally, she realized the source of her disappointment: guided inquiry gave her a far better understanding of what her students *didn't* know or understand. It made their lack of knowledge more apparent. With this better information, she found herself driven to develop new approaches to address students' misconceptions and concerns.

Among the most difficult issues for many teachers to rethink was assessment. How do you keep track of who knows what, when different students are pursuing different paths? How do

you judge performance? Can you quantify the kind of mathematical process knowledge that is at least as important as the factual knowledge of geometry? How do you weigh a bold conjecture in relation to an airtight proof?

In time the insistent quality of some of these increased demands receded. The technology began simply to be present as a classroom tool, in much the same way as the chalkboard or a shelf of reference books. Grading strategies emerged and were refined. Habits of classroom management developed. Students increasingly became accustomed to their new roles and responsibilities in the classroom.

Dilemmas

The constructivist approach to knowledge not only places new demands on teachers but also forces them to confront a series of new dilemmas.

The essence of this series of dilemmas is well captured in the work of Magdalene Lampert (in press) who points out the basic conflict between "What can I help these students learn?" and "What am I supposed to teach?" Let me spell out some of the implications of this conflict.

Teachers who adopt a constructivist approach often find it necessary to question the way they think about the very subject they are teaching. If the specific set of topics and the sequence in which they are covered is no longer completely prescribed, teachers begin to ask, "What is the purpose of teaching this material?" When they face the fact that not everyone in the class learns the same material, they must address the questions, "What is the essence of this subject?" and "Are there things about geometry that everyone should know?"

The pedagogical counterpart to this subject matter dilemma is the question of "how much structure?" On one hand, there must be sufficient guidance so that the process of inquiry is productive and fruitful and yields substantive mathematics. On the other hand, too much structure makes a mockery of inquiry and reduces classroom activity to the ceremonial rituals of traditional mathematics instruction. Should students completely reinvent geometry (see Healy, in press) or should inquiry, if desirable at all, be merely a motivator?

A second kind of dilemma arises because a constructivist approach is the enemy of "coverage." Inquiry is time consuming and may not fit neatly into the fifty-minute time bundles in which the school day comes packaged. If one commits oneself to taking students' ideas seriously and to following the implications and consequences of at least some of them, then life in the classroom becomes unpredictable, full of side trips and possible dead ends.

A constructivist approach is also at odds with the notion of problems that have unique correct answers. There are such problems. They are, however, not the real problems that face us in adult life. Nor, for that matter, are they the real problems of the disciplines we learn and teach in schools. If schools allow students to inquire, they are unlikely to pursue problems that have unique correct answers. The problem of assessment then demands delicacy, because "correctness" can no longer be the measure of how much and how well a student has learned the subject.

Many teachers who have adopted a constructivist approach in their teaching find themselves questioning their own identities as teachers. They begin to ask, "What does it mean to be a good teacher?" For some, the once adequate response of "making it through the lesson or the textbook" will no longer suffice. It can be unsettling when students leave the class after a lesson, a lab, or a week with no crisp sense of closure, completeness, or accomplishment. Teachers often begin to recast the question of what it means to be a good teacher so that it becomes, "Am I giving the students what they need?" This brings us back to the dilemma above, "What do students need from geometry?"

The teachers are not the only ones confronted with these dilemmas. "Coverage" is not solely a concern of teachers. Students in guided inquiry classrooms often compare their "progress" through the textbook, if they have one, with that of their peers in traditional classes and ask, "Are we behind?" Some report feeling different from their friends and unable to share homework.

Because schools tend to transmit information, rather than support the acquisition of knowledge, and because students tend to internalize a

notion of intellectual authority inherent in teacher and text, they often feel at a loss to establish "truth." Many students coming from traditional classes find the shift in their roles that accompanies guided inquiry disconcerting and the lack of single correct answers disorienting. Students will ask, "How do we know if it's right?" about some newly devised conjecture that emerges from the class. Only when they realize that knowledge is not the private preserve of teachers and textbook writers have they internalized the essential message of guided inquiry.

Some students are also relieved about what is to come when they move on to the next year's mathematics class. They look forward to algebra class, where they will "just learn like we did before. Right from the book and have the teacher teach." A return to normalcy.

Shifts in Thinking

The kinds of intellectual and psychological adjustments that many teachers and students have to make in order to become comfortable with a pedagogy of guided inquiry and with curricular materials like the *Geometric Supposer* are not minor changes, even for the best of teachers and students. The nature and extent of these changes stands in contrast to the kinds of changes we often ask those in schools to make. The adoption of a new text, or the addition of a new topic or even a new course to the curriculum, does not engender the same questioning of fundamental assumptions. What we have asked teachers and students to do is to face a range of profound questions about every aspect of learning and teaching in schools. We are talking about violating norms, shifting paradigms, and making a cultural transition. It is little wonder the process seems difficult.

The Implications

What, then, have we learned from the overall *Supposer* experience?

First, we have learned that technology can serve as a *catalyst* for change, but it can't carry the day. We believe that technology — with appropriate software, with appropriate materials, with an appropriate pedagogy, and with appropriate support for the learning community that is the classroom — can catalyze change. Specifically,

we believe that technology can play a role in empowering students to acquire knowledge on their own and to develop the concomitant sense of intellectual independence that is the hallmark of a successful education. Further, we believe that technology can help teachers to become sufficiently self-confident and self-assured that they will willingly abdicate their role as supreme intellectual authority and share such authority with those who earn it. Needless to say, these changes require working out a vastly different set of roles, rules, and relationships among teachers, students, texts, and subject matter. We hasten to emphasize that the technology, while necessary, is not sufficient. It is at best a tool. And like any tool, it can be abused and used in ways that mock its potential. We have seen *Supposer* classrooms in which a whole class was led through an "inquiry," keystroke by keystroke.

Second, we have learned that teachers are central to the constructivist approach and must be supported. If we want to take advantage of the constructivist approach to learning and teaching with technology, we must resolve to support teachers with adequate time and appropriate mechanisms for study and preparation, and with appropriate curricular materials. We must question whether the existing school structures allow for the kind of collegial interaction among teachers that we have come to believe is necessary for the success of this approach to education.

Moreover, we must help teachers to cope with the new and increased demands that will be placed on them, and to wrestle with the dilemmas with which they will inevitably be confronted.

Teachers by themselves, though, constitute neither the problem nor the solution. It is both unfair and naive to place the responsibility and burden of embracing and implementing such fundamental change solely on the shoulders of teachers. Classrooms and teachers do not exist in a vacuum. Rather, what happens in the classroom is the product of a much broader teaching and learning system that exists beyond as well as within the walls of our schools. If we want changes to occur, we must be prepared to undertake what Jerrold Zacharias, M.I.T. pro-

fessor, progenitor of school curriculum reform, and mentor to me and to many in the curriculum improvement business, called a proper systems engineering job.

What would it take? Like the MSEB and NCTM, we need to think about all the actors — from students and parents right on through communities and states to the President of the United States — who need to be engaged; and about all the tasks and problems, from curriculum and testing to equity and research, that must be tackled to make school reform a reality. We need to get our restructuring act together. Reforms like those of MSEB and NCTM, calling for restructuring content and instruction, must be integrated with reforms calling for the restructuring of schools and of the ways time and people in school buildings are organized, and with the reforms calling for restructuring the teaching profession. We need to take seriously the notion of Elmore and McLaughlin (1988) that successful reform and innovation require an understanding of the interaction of policy, administration, and practice. We need to heed Cohen's warning that inquiry-oriented instruction runs counter to "an old and deeply rooted scholastic inheritance" in which "teaching is telling, learning is accumulation, and knowledge is facts, strung together by rules of procedure" (1988, p. 256).

Ordinary sources of knowledge are at least as powerful an influence on children's learning as their school instruction. And in large parts of America, this ordinary knowledge is quite traditional and closely tied to the instructional practices that inquiry-oriented approaches to reform have sought to replace. (p. 263)

Clearly teachers and students, technology and texts are not sufficient.

We have no choice but to undertake it all. The price will be high, but the price of not doing so will be higher still.

What Is the *Geometric Supposer* a Case Of?

Let me conclude with an observation. Nowhere in the previous section on the implications of what we have learned is the word "mathematics" mentioned. That is neither an oversight nor a coincidence. We believe that what we have learned from the *Supposer* experience about

reforming mathematics education can and must apply more broadly to education in general. Like technology and teachers, neither geometry by itself, nor even mathematics by itself, is up to the task.

We want questioning, intellectually independent students in all subject areas. We want flexible teachers in every discipline, who can be both members of and managers of learning communities. We want schools to become knowledge-making places across the curriculum.

We will confront the same problems in all subject areas and at all levels of education. There will be new and more extended demands on teachers. There will be dilemmas to face. There will be a need to rethink roles and responsibilities for both students and teachers. And there will be systemic questions to consider.

The *Supposer* experience is not a clear model for how to bring about such change in education across the board, but the experience does shed light on the challenge and the opportunity.

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Notes

1 The *Geometric Supposer* was designed by Judah Schwartz and Michal Yerushalmy, was developed at the Center for Learning Technology of Education Development Center (EDC), and is published by Sunburst Communications, Inc. In addition to the software, EDC has developed collections of exercises, curriculum units, videotapes, and a training manual to support the use of the programs in the classroom.

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